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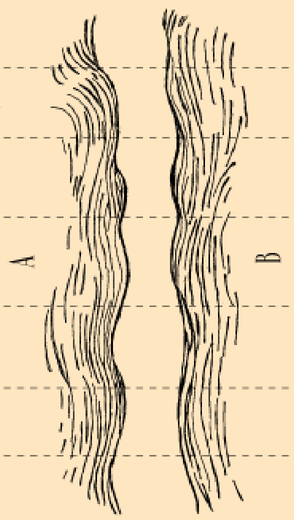

Ioannis M. VANDOULAKIS, « The Arbitrariness of the sign in Greek Mathematics »

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The Arbitrariness of the Sign in Greek Mathematics

Ioannis M. Vandoulakis

Abstract. In this paper, we will examine some modes of signification of mathematical entities used in Greek mathematical texts in the light of Saussure's conceptualization of sign. In particular, we examine certain mathematical texts from Early Greek period, the Euclidean and Neo-Pythagorean traditions, and Diophantus.

0. Introduction

Greek mathematicians have used a wide variety of modes of signification to express mathematical entities. By *signification* here, we understand the relation between the form of the sign (the *signifier*) and its meaning (the *signified*), as used in Ferdinand de Saussure's semiology. According to Saussure, this relation is essentially arbitrary, motivated only by social convention. Moreover, for Saussure, signifier and signified are inseparable. One does not exist without the other, and conversely, one always implicates the other. Each one of them is the other's condition of possibility.

In this paper, we will examine some modes of signification of mathematical entities used in Greek mathematical texts in the light of Saussure's conceptualization of sign. In particular, we examine certain mathematical texts from the following periods and traditions:

- Early Greek mathematics: texts ascribed to Hippocrates of Chios as transmitted by Simplicius.
- "Golden Age" of Greek mathematics: Euclid's *Elements* and the works of geometers of 3rd century BC.
- Intermediate Period: texts of Nicomachus of Gerasa (c. 60 – c. 120 CE) and other Neo-Pythagorean authors.
- "Silver Age" of Greek mathematics[1]: Diophantus' *Arithmetica*.

1. Signs in Early Greek Mathematics

Hippocrates of Chios (c. 470 – c. 410 BCE) is reported by Proclus to have been the first to write a systematically organized geometry textbook, called *Elements*. Only a single fragment of Hippocrates' work survived, embedded in the work of Simplicius (c. 490 – c.

[1] The name of this era that follows a period of stagnation after Ptolemy, i.e. the period between 250 and 350 AD., belongs to Boyer "Revival and Decline of Greek Mathematics" [Boyer (1991) [1989], 178].

560), where the area of some *Hippocratic lunes* is calculated.

In this fragment, we meet a kind of signification of geometric objects that is not common in other extant works of Greek mathematicians. Specifically, a point or line-segment or a figure is denoted by the following locutions: “the point on which A stands” (or, “is marked by A”) (τὸ ἐφ’ ᾧ A); “the line on which AB stand” (or, “is marked by AB”) (ἡ ἐφ’ ᾗ AB); “the trapezium on which EKBH stands” (or, “is marked by EKBH”) (τὸ τραπέζιον ἐφ’ οὗ EKBH) (see Figure 1)

> ἔστω κύκλος οὗ διάμετρος ἐφ’ ᾗ [ἡ] AB, κέντρον δὲ αὐτοῦ ἐφ’ ᾧ K. καὶ ἡ μὲν ἐφ’ ᾗ ΓΔ δίχα τε καὶ πρὸς ὀρθὰς τεμνέτω τὴν ἐφ’ ᾗ BK· ἡ δὲ ἐφ’ ᾗ EZ κείσθω ταύτης μεταξὺ καὶ τῆς περιφερείας ἐπὶ τὸ B νεύουσα τῶν ἐκ τοῦ κέντρου ἡμιολία οὖσα δυνάμει. ἡ δὲ ἐφ’ ᾗ EH ἤχθω παρὰ τὴν ἐφ’ ᾗ AB. [Bulmer-Thomas, 1939, I, 242-244].
>
> “Let there be a circle with diameter marked by AB and center marked by K. Let the [straight line] marked by ΓΔ bisect the other one marked by BK at right angles; and let the [straight line] marked by EZ be placed between this and the circumference verging towards B, so that the square on it is one-and-a-half times the square on one of the radii. Let the [line] marked by EH be drawn parallel to the other one marked by AB” [my emphases].

In this text, the letters used do not actually *name* geometrical objects (the point, the line segment or the trapezium, respectively), but serve as *markers* or *indicators* to label or indicate *concrete* geometrical objects. Thus, for Hippocrates, AB is not the name of diameter. AB is a visible sign pattern to point to the diameter. In other words, AB is a “label”, pointing to the diameter in Figure 1. Therefore, letters in Hippocrates are signs that show (spatial) evidence of the object being signified. [Vandoulakis, 2018].

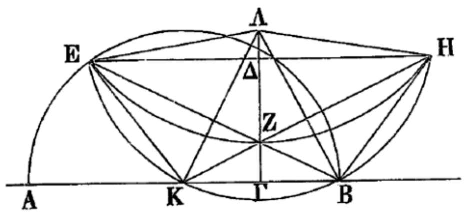

Figure 1

This kind of signification is close to Morris’ concept of *identifior*. This concept corresponds to Peirce’s *index* [Morris 1971, 154, 362], but in contrast to Peirce, Morris’ identifior is restricted to spatio-temporal *deixis*, i.e. an identifior indicates a location in space (*locatum*) and directs the reader toward a certain region of the environment. According to Morris, the identifior.

> “has a genuine, though minimal, sign status; it is a preparatory-stimulus influencing the orientation of behavior with respect to the location of something other than itself”. [Morris 1971, 154]

Morris further distinguished a subclass of identifiors, called *descriptors*, which describe a spatial or temporal location. This is what we face in Hippocratic text.

2. Signs in Euclid’s arithmetic

A more complicated semiotic picture is found in the arithmetical Books of Euclid’s *Elements*. Euclid makes a distinction between the concept of number-*arithmos* (ἀριθμός) and the concept of “multitude” (πλῆθος). The former is a whole made up of units and signified by a line segment. The latter is neither given a name, nor ever signified by any sign. [Euclid (Stamatis), 1969-77]. It is not an *arithmos*, according to the Euclidean definition. It is a mental signifier, which expresses the iterative step in the generation of number, i.e. the number of units contained in the corresponding multitude.

Euclid constructs his arithmetic for numbers-*arithmoi*, that is for the numbers signified as segments, while the arithmetic of multitudes is taken for granted. Accordingly, arithmetic is constructed as a formal theory of numbers-*arithmoi*, while the concept of *multitude* or iteration number has a specific meta-theoretical character [Vandoulakis 1998].

Consequently, the Euclidean number-*arithmos* has the following formal structure:

$$A\ f \quad \{aE\}_{a\geq 2}$$

where E signifies the unit and a is the number of times that E is repeated to obtain the number-*arithmos A*.

Euclid’s use of letters in elaborating arithmetic is functional. This is made evident by the fact that Euclid uses two ways to signify numbers. Numbers are signified by one letter standing on a segment, or by two letters standing on the extremes of a segment, depending on the expressive requirements of the proof [Papadopetrakis 1990]. The signification by two letters is used when certain operations on numbers are going to be done in the process of the proof, such as addition, subtraction of segments, or division of a segment into subsegments.

Enunciations state some general property about numbers-*arithmoi*, where the word ἀριθμός is used without article. Further, Euclid proceeds to the *ekthesis* of the proposition, where he introduces numbers-*arithmoi* by means of line segments signified by one or two letters; now the word ἀριθμός is used with the definite article standing before it and the number is specified, although indefinite. In this way, general statements about numbers are interpreted as statements about an arbitrary given (indicated) number. In virtue of the substitution described above, the process of proof takes places actually with an arbitrarily given number. After the proof of the statement about the specified number is done, the conclusion is reverted to the enunciation and it is claimed that the statement has been proved for the general case.

In this way, arithmetical propositions that are proved for a segment or a finite configuration of segments are considered as proved for *any* segment, i.e. the statement holds generally. The particular physical characteristics of the diagrams of figures are taken to be irrelevant.

3. Signs in various contexts in Antiquity

When the Euclidean mode of symbolism of mathematical objects by segments was established, the usage of the locutions έφ’ ᾧ, έφ’ οὗ and the like was not abandoned. For instance, in Aristotle’s discussion of Zeno’s paradoxes, that is in a non-mathematical context, going back to the Pre-Socratics, we face a very peculiar use of these locutions. The associated concrete objects (figures), on which the letters are supposed to stand, are missing.

> ἔστω τὸ μὲν έφ’ ᾧ Α θᾶττον, τὸ δ’ έφ’ ᾧ Β βραδύτερον, καὶ κεκινήσθω τὸ βραδύτερον τὸ έφ’ ᾧ ΓΔ μέγεθος έν τῷ ΖΗ χρόνῳ. [Aristotle (Bekker), 1960, *Physics* VII 232b 27-29]
>
> Let the one marked by A be the quicker, and the other marked by B the slower, and let the slower has traversed the magnitude marked by ΓΔ in the time ZH [my translation].

The moving objects A and B and the traversed distance ΓΔ during the time interval ZH are not indicated but should be imagined or intented. Here, the identifiors are used to signify intented spatial and temporal location.

This kind of signification is close to Prieto’s understanding of *indices* [Prieto 1966]. Prieto defines an indice as any immediately perceptible fact that sheds light on a fact that is not immediately perceptible. Thus, indices are not *significant*, but are *significative*, in the sense that they come to mean something to the observer through a process of interpretation.

The locutions έφ’ ᾧ, έφ’ οὗ and the like were also used by the geometers of the late antiquity, notably Archimedes, to signify a specific geometrical object in a complex figure, for instance a conic, a spiral, etc. Thus, Archimedes uses these locutions to refer to a conic

(Figure 2):

> Ἔστω γὰρ ὀξυγωνίου κώνου τομά, ἐφ' ἇς τὰ Α, Β, Γ, Δ, διάμετρος δὲ αὐτᾶς ἁ μὲν μείζων ἔστω, ἐφ' ἇς τὰ Α, Γ, ἁ δὲ ἐλάσσων, ἐφ' ἇς τὰ Β, Δ, ἔστω δὲ κύκλος περὶ διάμετρον τὰν ΑΓ. [Archimedes (Heiberg), *On conoids and spheroids*, 2013, 1, 306-8]
> Let a section of acute-angled cone [i.e., an ellipse], marked by Α, Β, Γ, Δ, and let the major axis be marked by Α, Γ, and the minor axis be marked by Β, Δ, and let a circle of diameter ΑΓ [my translation].

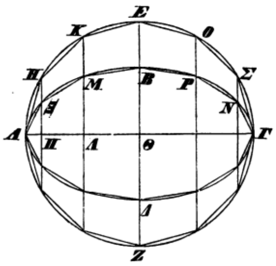

Figure 2

We find the same way of signifying a conic in Apollonius (Figure 3).

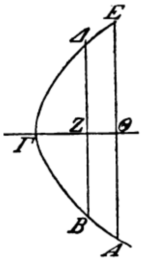

Figure 3

> ἔστω ἡ δοθεῖσα κώνου τομή, ἐφ' ἧς τὰ Α, Β, Γ, Δ, Ε σημεῖα. δεῖ δὴ αὐτῆς τὴν διάμετρον εὑρεῖν. [Apollonius (Heiberg), *Conics*, Book 2, Section 44, line 2].
> Let a given section of cone marked by the points Α, Β, Γ, Δ, Ε. It is required to find its diameter [my translation].

In his work *On Spirals*, Archimedes uses similar signification to label a spiral (Figure 4).

> Ἔστω ἕλιξ, ἐφ' ἇς τὰ Α, Β, Γ, Δ, ἔστω δὲ ἀρχὰ μὲν τᾶς ἕλικος τὸ Α σαμεῖον, ἀρχὰ δὲ τᾶς περιφορᾶς ἁ ΑΔ εὐθεῖα, καὶ ἐπιψαυέτω τᾶς ἕλικος εὐθεῖά τις ἁ ΖΕ. [Archimedes (Heiberg), *On Spirals*, 2013, 2, 56].
> Let a spiral marked by Α, Β, Γ, Δ, and let the point Α be the starting-point of the spiral and the straight line [i.e. the ray] ΑΔ be the starting [position] of the circuit and a straight line ΖΕ tangent to it [my translation].

Here, the first circuit of a spiral line is marked by the letters Α, Β, Γ, Δ, whereas the starting point Α, and the ray ΑΔ are named properly.

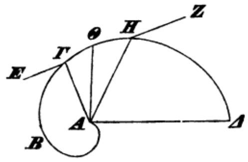

Figure 4

In all these instances, the letters are used to indicate (or label) a part of an (infinite) figure (usually a line, other than a straight line or a circle) in the drawing, but not to name a geometrical object. This kind of signification is an identifier, in Morris' sense, i.e. it signifies a location in space (*locatum*) and directs the reader (*deixis*) toward the appropriate part of the figure.

4. Signs in the Neo-Pythagorean arithmetical tradition

Another kind of symbolism of visual-type is used in Nicomachus' Introduction to Arithmetic. Numbers are designated by means of letters by *convention* (νόμῳ), not by *nature* (ού φύσει). The natural *semeiosis* (φυσική σημείωσις) of numbers is signified by means of the representation of the units composing a number, one beside the other.

> First, however, we must recognise that each letter by which we designate a number, such as iota, the sign for 10, kappa for 20, and omega for 800, signifies that number by man's convention and agreement, not by nature. On the other hand, the natural, unartificial, and therefore simplest designation of numbers would be the setting forth one beside the other of the units contained in each. [Nicomachus (Hoche) 1866, II. vi, 2; Nicomachus (D'Ooge) 1926, 832].

The concepts of *convention* (νόμος) and *nature* (φύσις) go back to the pre-Socratics (Pythagoras, Democritus) and the Sophists. Pythagoras is reported to have shared a 'naturalistic' view, i.e. that the assignment of names to things is not an arbitrary operation, but is imposed upon things by some kind of *natural* adequacy between the names and the things, so that

> "The activity of naming, then, according to Pythagoras, belongs not to any random individual but to one who sees the Intellect and the nature of the real entities. Names are therefore natural." [Proclus (Pasquali) 1908, 16, p. 5, 25; Proclus (Duvick) 2007, 14].

The original 'naturalistic' viewpoint is also held by Iamblichus, who accuses Philolaus of having abandoned the master's viewpoint and adopted the 'conventionalist' view. The opposition between the 'naturalistic' and the 'conventionalist' semantic viewpoints is the point of departure in Plato' *Cratylus*, where semantic conventionalism is attributed to Hermogenes, while semantic naturalism is supported by Cratylus. Proclus ascribes to Democritus the view that the relation between names and things named is *conventional*, rather than natural [Kretzmann 1967, 359-361].

Thus, number in Nicomachus possesses internal structure (σχῆμα - 'arrangement'). It is a (finite) 'suite' (or a schematic pattern) of signs, unbounded in the direction of increase and bounded below by the *monas* in the direction of decrease.

Further, the finite sequence of such simple 'suites' can be constructed, i.e. the sequence of the so called "properly ordered" (εὐτάκτους) numbers

α, β, γ, δ, ε, ς, ζ, η, θ, ι, ια, ιβ, ιγ, ιδ, ιε, ...

i.e. is the sequence (which we denote by natural numbers in italic)

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, …

which is called *the natural suite* (ὁ φυσικός στῖχος) by Nicomachus [Nicomachus (Hoche) 1866, II, viii, 3] and serves as a pattern exemplifying the mode of construction of the kind of number considered in each case[2]. [Vandoulakis, 2010].

5. Signs in Diophantus' *Arithmetica*

Although Diophantus proceeds in his *Arithmetica* from the Euclidean definition of number as a collection of units (i.e. from the natural numbers), in the investigation of problems he searches for positive rational solutions, which he calls also "number" (ἀριθμός) and designates by the sign ϛ. In other words, he extends the concept of number to the whole set of positive rational numbers, by integrating the unknown into numbers.

Further, he introduces *literal signification* for the powers of the unknown. Specifically, he introduces special signs for the first six positive powers of the unknown, the first six negative powers, and for its zero power, following the *additive principle* in the formation of the literal signs.

δύναμις (power)	Δ^{Y}	Square of the unknown (x^2)
κύβος (cube)	K^{Y}	Cube (x^3)
δυναμοδύναμις (power-power)	$\Delta^{Y}\Delta$	Fourth power (x^4)
δυναμοκύβος (power-cube)	ΔK^{Y}	Fifth power (x^5)
κυβοκύβος (cube-cube)	$K^{Y}K$	Sixth power (x^6)

[Diophantus (Tannery), 1893-95].

The negative powers are designated by using the sign χ, i.e. by $\Delta^{Y\chi}$ is designated what we today denote by x^{-2}. The zero power of the unknown is designated by $\overset{\circ}{M}$, which is not identical to number 1, but is understood as the side of a square number. He also defines a 'multiplication table' for the powers of the unknown by a rule that could be succinctly be written in modern notation,

[2] The natural suite should not be confused with the natural series. In Nicomachus, it is a finite constructional element. The concept of the natural suite is intrinsically connected with the notion of "proper order." In Nicomachus, the natural suite is always a suite of "properly ordered" numbers, i.e. it embodies the specific regularity or rule according to which it is constructed.

$$x^{m} \cdot x^{n} = x^{m+n} \qquad\qquad x^{m} \cdot \frac{1}{x^{n}} = x^{m-n}$$

where $|m| \leq 6$, $|n| \leq 6$, $|m+n| \leq 6$. It is noteworthy that neither the 4th, 5th, and 6th powers, nor the negative powers of the unknown could be geometrically visualized.

Moreover, he introduces the symbol $\pitchfork$ (inverted Ψ, 'psi') to designate the minus sign. For equality, he introduces the sign ἴσ by using the first two letters from the word 'ἴσος', which means 'equal'. In contrast to the signs that signify powers of the unknown, i.e. numbers, the signs for minus and equality do not signify numerical objects in any sense, but abstract concepts. On the other hand, the sign Ω is used to designate an indeterminate square. Although the latter can be considered as iconic sign, i.e. a picture of a square, the designatum is an abstract object, namely an indeterminate square.

Using this system of signs, Diophantus was able to construct 'words' out of these signs that represent *equations* in literal form. For instance, the 'words' $K^{Y}\overline{\alpha}$ ἴσ $\overset{o}{M}\overline{\beta} \pitchfork$ ϛ $\overline{\alpha}$, $K^{Y}\overline{\alpha\iota} \pitchfork \Delta^{Y}\overline{\beta}\overset{o}{M}\overline{\alpha}$ ἴσ $\overset{o}{M}\overline{\varepsilon}$ signify the equations that can be written today as $x^{3} = 2 - x$ and $x^{3} - 2x^{2} + 10x - 1 = 5$, respectively. Concrete numbers are designated by the letters of the Greek alphabet with bars over them, i.e. $\overline{\alpha}, \overline{\beta}, \ldots, \overline{\theta}$ designate the numbers from 1 to 9; the next eight letters $\overline{\iota}, \overline{\kappa}, \ldots, \overline{\pi}$, and the koppa $\overline{\&}$ or $\overline{\%}$ designate the multiples of 10 from 10 to 90; the last eight letters and the sampi $\overline{"}$ designate the multiples of hundreds from 100 to 900.

In spite of the geometric language that Diophantus uses (i.e. 'side', 'square', 'cube', etc.) in forming equations (e.g. 'add a square', 'cube', 'side', etc.), he treats the designated objects as numbers. Furthermore, this kind of literal symbolism was used not only to write *equations*, but also to manipulate them and solve indeterminate equations and systems of indeterminate equations up to 6th degree in rational numbers. In particular, in his "Introduction" he formulates two rules of transformation of equations:

a. The rule for transfer of a term from one side of an equation to the other with changed sign, and
b. The reduction of like terms.

These rules became known under their Ababized names of *al-jabr* and *al-muqābala*.

Diophantus examines even indeterminate equations in more than one unknown, where the additional unknowns are expressed as linear, quadratic, or more complex rational functions of the first unknown and uses concrete values, for the first time, to designate *parameters* [Bashmakova and Slavutin, 1984]. In order to solve a problem, Diophantus represented the required numbers as rational functions of a single unknown and of parameters.

He assigned to the parameters concrete numerical values but stipulated that *these could be replaced by other **arbitrary** rational numbers*, or by ***arbitrary*** rational numbers satisfying certain conditions.

As an illustration, we consider problem 8 in book 2:

To divide a given square number into two squares [Diophantus (Heath), 1885, 144].

i.e., in modern terms, to solve the equation

$$x^{2} + y^{2} = a^{2}$$

Diophantus starts from the arbitrarily given square number 16, i.e. $a^2 = 16$. Further, his line of reasoning can be exposed, in modern terms, as follows: he takes the base of one of the squares as the unknown $x = t$, and the base of the other square as a linear function of *t*:

$$y = kt - 4,$$

or, in Diophantus' words,

"I form the square from any number of ἀριθμοί … *minus* as many units as there are in the side of 16 [i.e. 4]". [Diophantus (Heath), 1885, 144 footnote 1].

Here 4 is a root of 16 and *k* can be an arbitrary rational number. The formulation "*any* number of ἀριθμοί" clearly expresses the fact that the parameter *t* is arbitrary. Accordingly, Diophantus' version of equation can be expressed, in modern notation, as follows:

$$t^2 + (2t - 4)^2 = 4^2.$$

The solution of the problem is given, in modern symbolism, by

$$x = t = \frac{2ak}{1 + k^2}$$

$$y = kt - a = a\frac{k^2 - 1}{k^2 + 1}$$

In Diophantus' case, $x = 16/5$ and $y = 2t - 4 = 12/5$.

However, Diophantus does not confine himself to a single solution. He seems aware of the fact that for any rational *k* one obtains a corresponding rational solution. This becomes clear in problem 19 of book III, where he clarifies that

"we saw how to divide a square into two squares in an infinite number of ways." [II.8] [Diophantus (Heath), 1885, 166].

Thus in problem 8 of book II, number 2 performs two distinct functions:

a) that of the concrete number 2, and
b) that of a sign, which *stands for* an ***arbitrary*** rational number.

However, it is not always possible for a parameter to assign a convenient arbitrary value. In this case, Diophantus sets forth additional conditions. Let us consider, for example, the problem 8 of book 6, which is expressible, in modern symbolism, by the system of equations

$$x_1^3 + x_2 = y^3,$$

$$x_1 + x_2 = y.$$

He starts by putting, in modern terms, $x_2 = t,\ x_1 = kt$, where, in Diophantus' case, $k = 2$. Then, from the second equation we obtain $y = (k+1)t$, and from the first one,

$$t^2 = \frac{1}{(k+1)^3 - k^3}.$$

For $k = 2$ we obtain $t^2 = 1/19$, i.e., *t* is not rational. In order to obtain a rational solution, the way that t^2 is expressed in terms of the parameter *k* is examined. Since the expression in question is a fraction with numerator 1, which is a square, the denominator must also be a square, i.e. $(k+1)^3 - k^3 = \mathrm{W}$. As the new unknown is taken $k = \tau$ (designated by the same sign as the original unknown x_2); hence,

$$(\tau+1)^3-\tau^3=\mathrm{W}$$

or

$$3\tau^2+3\tau+1=\mathrm{W}\ .$$

By putting

$$\mathrm{W}=(1-\lambda\tau)^2$$

we obtain

$$\tau=\frac{3+2\lambda}{\lambda^2-3}.$$

By choosing $\lambda=2$, we obtain $\tau=7$. Hence, the value of the parameter can be chosen from the class of numbers

$$\left\{\frac{3+2\lambda}{\lambda^2-3}\right\}.$$

Then, Diophantus goes back to solve the original problem.

Consequently, we see that in Diophantus' sign system, in addition to the signs for the unknown and its powers, a major role is played by *concrete number symbols,* which stand also for *parameters*. In the latter case, they can play the role of *free parameters* or of *non-free parameters* satisfying certain supplementary conditions.

Conclusion

In Greek mathematics, we observe a wide diversity in the use of signs signifying mathematical objects. In early Greek mathematics, signs are used as identifiors to direct the reader toward a certain region of the figure. This way of signifying mathematical objects continues to be used during the Hellenistic era for signifying a specific (infinite) geometrical figure (other than a straight line or a circle), in a complex drawing. These uses of signs do not name geometrical objects.

In the Neo-Pythagorean arithmetical tradition, we find explicit evidence that signs signify numbers by convention, not by nature. This use of signs for numbers apparently goes back to the 5th century BC, when the controversy between semantic conventionalism and naturalism made its appearance.

In Euclid's *Elements*, numbers are signified by a line segment, made up of a *multitude* of units; multitudes are neither named nor signified by any sign, but have a specific meta-theoretical character. General statements about numbers are interpreted as statements about an arbitrary given (indicated) number, so that proof takes places actually with an arbitrary given number.

An elaborate system of signs is found in Diophantus' *Arithmetica*, where *literal signification* for the powers of the unknown is introduced, enabling the construction of complex signs signifying indeterminate equations. These signs are operational, since they facilitate the transformation of equations. The most advanced feature of Diophantus sign system is the use of concrete numerical values as *parameters*, i.e. as arbitrary positive rational numbers or as arbitrary rational numbers satisfying certain conditions.

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